*Multiplexity*

Multiplexity is the tendency for ties of different kinds to occur together, for example, for siblings to help with the rice harvest. Multiplexity is summarized for pairs of relations by focusing on the configuration of ties in the pair on two relations simultaneously. For each ordered pair of households (i, j), and two relations A and B, there are four possibilities: i has ties to j on both relations A and B; i has a tie to j on relation A but not on relation B; i has a tie to j on relation B but not on relation A; i has ties to j on neither relation. These possibilities can be summarized in a 2 by 2 table:

| | i -->j on Relation B | |
|---|---|---|
| i --> j on Relation A | yes $x_{ijB} = 1$ | no $x_{ijB} = 0$ |
| yes $x_{ijA} = 1$ | a | b |
| no $x_{ijA} = 0$ | c | d |

Each of the n(n-1) ordered pairs of households is counted in one of these cells. The “a” cell indicates both ties are present. (The “a”, “b”, “c”, “d”, notation is common in describing measures of similarity in 2 by 2 tables).

We use three approaches to multiplexity: a match coefficient, referred to as Russell and Rao’s coefficient, the Pearson correlation, and a count of the number of matches along with the mean (expected number of matches) and variance for the number of matches along with z-score comparing the observed number of matches to the expected number.

1. The Russell and Rao coefficient (sometimes referred to as a positive match coefficient) is defined as the number of pairs where both $x_{ijA} = 1$ and $x_{ijB} = 1$, divided by the total number of pairs. This is a descriptive measure that takes on values from 0 (when the two relations never occur together) to 1, where both relation always occur together. In terms of the notation for cells in the 2 by 2 table it is equal to:

$$\frac{a}{(a+b+c+d)}$$

Alternatively, using notation for entries in the sociomatrix, it is equal to:

$$\frac{\sum x_{ijA}\ x_{ijB}}{n\times(n-1)}$$

where n is the number of actors (households) in the network.

2. The Pearson correlation coefficient (sometimes referred to as Pearson's phi for the two by two table) is a second descriptive measure of multiplexity. It ranges from -1 to +1. In terms of the notation for cells of the 2 by 2 table, it is equal to:

$$\frac{(ad)-(bc)}{\sqrt{(a+b)\times(c+d)\times(a+c)\times(b+d)}}$$

(see Gower, John "Measures of similarity, dissimilarity, and distance." in *Encyclopedia of Statistical Science*.) Standard statistical tests for the Russell and Rao and Pearson coefficients do not apply because the observations (the n(n-1) pairs of actors in the network) are not independent.

3. The third approach to multiplexity follows the logic of randomization / permutation tests for comparing matrices, first introduced by Mantel (1967) and developed by Hubert and colleagues. This approach to multiplexity calculates the mean and variance for the number of matches (the count of the number of pairs for which ties on both relations are present $x_{ijA} = 1$ and $x_{ijB} = 1$) and then calculates a z-score for the observed number of matches. (Baker and Hubert 1981; Hubert and Baker 1978). The general idea is to construct a permutation distribution to evaluate the degree of similarity of two matrices. One matrix (A) is "fixed" while the rows and simultaneously the columns of the other (B) are randomly permuted. For each permutation a measure of similarity of the two matrices is calculated (the measure we use is the number of matches ). The permutation distribution is the distribution of the calculated indices over several hundred or thousands of random permutations. The obtained index of similarity for the two observed sociomatrices is then referenced to this permutation distribution. The obtained index is significantly large (or small) if it falls in the upper (lower) α% of the permutation distribution. This approach is often referred to as the "quadratic assignment procedure" or "QAP" for short. (Baker and Hubert 1981; Hubert and Baker 1978). Rather than constructing the permutation distribution, we use the exact moments of the permutation distribution (mean and variance) using equations in Baker and Hubert (1981) and Hubert and Baker (1978). We then calculate a z-score for the observed index.

References:

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